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Issued on Sep 1, 2017
Solutions due: Sep.7, 2017

CS422

Fall 2017, Assignment #1

PROBLEM 1 (1P+2P+2P+1P) **:**
For $n \in \mathbb{N}$ let $\ell(n)$ denote the least number of multiplications to compute the monomial x^n from x.

a) Recall why it holds $\log(n) \leq \ell(n) \leq 2\log(n)$ for all $n \geq 2$, where $\log(n) := \log_2(n)$.

b) Describe an algorithm asserting $\ell(2^{16}-1) \leq 19$.

c) Fix $k,d \in \mathbb{N}$ and $a_0,\ldots,a_d \in \{0,\ldots,2^k-1\}$ and prove

$$\ell\big(a_0 + a_1 \cdot 2^k + a_2 \cdot 2^{2k} + \cdots + a_d \cdot 2^{d \cdot k}\big) \;\leq\; 2^k + (k+1) \cdot d$$

d) Improve (a) by showing $\ell(n) \leq \log(n) + \mathcal{O}\big(\frac{\log(n)}{\log\log(n)}\big)$.

Hint: In (c) choose $d := \lceil 1 + \log(n+1)/k \rceil$ and $k :\approx \log\log(n) - 2\log\log\log(n)$.

PROBLEM 2 (1P+2P) **:**
Devise a finite automaton for the following languages.

a) $\big\{\vec{x} \in \{\mathtt{0},\mathtt{1},\mathtt{2}\}^* \;\big|\; \vec{x} \text{ represents the ternary expansion of an integer divisible by } 3\big\}$

b) $\big\{\vec{x} \in \{\mathtt{0},\mathtt{1}\}^* \;\big|\; \vec{x} \text{ represents the binary expansion of an integer divisible by } 3\big\}$.

(most significant digit first)

PROBLEM 3 (1P) **:**
Does there exist an algorithm for deciding whether God exists? Justify!