

CS422

Fall 2017, Assignment #5

PROBLEM 11 (1+1+1+2+3+2+1P) :

A Boolean term $t(x_1, \dots, x_n)$ in 2CNF induces the (so-called *implication*) graph G_t with literals $x_1, \dots, x_n, \neg x_1, \dots, \neg x_n$ as vertices and directed edges $(\neg u, v)$ and $(\neg v, u)$ between literals u, v for every clause $(u \vee v)$ of t .

- Prove: If t has a satisfying assignment and x is a variable of t , then G_t admits no path from x to $\neg x$ or no path from $\neg x$ to x .
- Suppose conversely for each variable x in t that G_t admits no path from x to $\neg x$ or no path from $\neg x$ to x . Conclude that t has a satisfying assignment.
- Show $2SAT \in \mathcal{P}$.
- Consider the following problem called **Equivalence**, and prove it $\text{co}\mathcal{NP}$ -complete:

Given two n -variate Boolean terms s and t , does it hold $\forall \vec{a} \in \{0, 1\}^n : s(\vec{a}) = t(\vec{a})$?

- Consider the following problem called **MinBF**, and prove it $\mathcal{NP}^{\mathcal{NP}}$ -complete:

Given a Boolean term $t(\vec{x})$, is there a strictly shorter term $s(\vec{x})$ equivalent to t ?

- Prove that the following problems is \mathcal{PSPACE} -complete:

$$\left\{ \langle \mathcal{A}, \vec{x}, 2^N \rangle : \text{WHILE+ program } \mathcal{A} \text{ accepts input } \vec{x} \text{ in space } \leq N \right\}$$

- Inspired by (f), design a problem that is **EXP**-complete (with proof).

PROBLEM 12 (1+1P):

- Install the *public-key* system `pgp` on your computer; free versions are available from GNU for LINUX, WINDOWS, and MACOS X. Become familiar with the software (RTFM). Create a key pair! Deliberate on where to store the private and how to distribute the public part.
- Print, and submit on Nov.17, 'tickets' showing your name and public key's *fingerprint*. Email me your solutions of Problem #11 (scan/readable photo/PDF) by 9am of Nov.17, signed with your private key and encrypted with my public one: available for instance from <http://pgp.mit.edu/pks/lookup?op=get&search=0x227F4D274A4BE6FE> with fingerprint AF37 ECD4 AEBE 3D4E 76EB 4445 227F 4D27 4A4B E6FE.