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Solutions due: Mar.30, 2018

CS500

Spring 2018, Assignment #2

PROBLEM 2 (2+2+2+2+1P)**:**

a) Recall the two inductive definitions of a Binomial tree of order k:

   i) B_{k+1} is a B_k to whose root another B_k is attached as child.

   ii) B_{k+1} is a root with children $B_k, B_{k-1}, \cdots, B_1, B_0$.

   Prove by structural induction that the two definitions are indeed equivalent.

b) Prove that any binomial heap of N elements contains at most $\log(N)$ binomial trees.

c) Construct binomial heaps where both `ExtractMin` and `DecreaseKey` take time $\Omega(\log n)$.

d) Argue that the running time of a sequence of n calls to `InsertKey` is $\mathcal{O}(n)$, not $\Omega(n \log n)$.

e) Compare the binary heap and the binomial heap data structure:
   Describe in a table the operations they support, and in which running times.

PROBLEM 3 (2+2P)**:**

a) Prove: $\sum_j j \cdot q^j = q/(1-q)^2$ whenever $|q| < 1$.

b) Prove $1 + F_1 + F_2 + ... + F_k = F_{k+2}$, where F denotes the Fibonacci numbers.

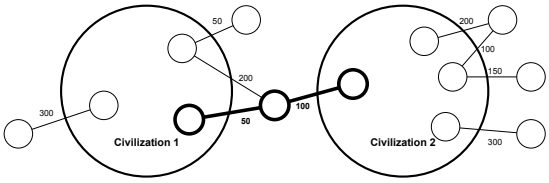
PROBLEM 4 (1+2P)**:**

a) Define a *Tri*nomial tree of order $k+1$ to be a T_k to whose root another *two* T_k are attached as children. How many nodes does T_k have?

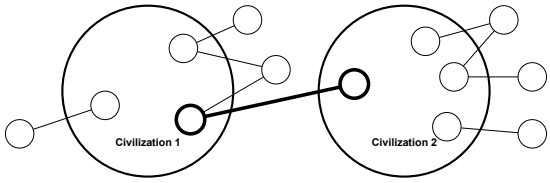
b) Define *Tri*nomial heap similarly to Binomial heap. Describe its heap operations and show their worst case costs.

PROBLEM 5 (4P)**:**

Devise and analyze an algorithm to solve the following problem:
You are a software engineer developing Sid Meier's Civilization VII. In this game, the battlefield is represented by a graph $G = (V,E)$ with a cost function c. The AI has a simple strategy. Each civilization $1 \leq i \leq n$ starts at a starting vertex $s_i \in V$. In each turn, each civilization i choose an edge $e \in E$ which incident to its territory with the minimum cost $c(e)$; then, conquers the other end of e. If two civilizations try to conquer the same vertex or become neighbors by an edge, they must start a war and the winner takes the entire territory of the other. You may assume the winner is decided arbitrarily. If only one civilization lasts, the game is over.
You are supposed to devise an algorithm to simulate a game with the strategy and analyze the running time of the algorithm in terms of $|V|, |E|$, and n.



(a) Civilization 1 and 2 try to conquer the same vertex.



(b) Civilization 1 and 2 try to conquer the same vertex.