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CS204

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Problem 1. *1 + 1 + 1 + 2 pts*

For each of these items, prove or disprove that the function is bijective:

a) $f : \mathbb{R}^+ \to \mathbb{R}$, $f(x) = \log x$
b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = e^x$
c) $f : \mathbb{Z}^2 \to \mathbb{Z}^3$, $f(a, b) = (a + b, a + 2b, a + 3b)$
d) $f : \mathbb{R}^+ \times [0, 2\pi) \to \mathbb{R}^2 \setminus \{0\}$, $f(r, \theta) = (r \cos\theta, r \sin\theta)$

Problem 2. *1 + 1 + 1 + 2 + 2 pts*

Compute each of these items. Show your calculation process.

a) $\sum_{i=1}^{100}(1 + (-1)^i)$
b) $\sum_{i=1}^{10}(2^{i+1} - 2^i)$
c) $\sum_{i=1}^{10}\sum_{j=1}^{10} ij$
d) $\sum_{n=1}^{100} \frac{1}{n(n+1)}$ (Hint: $\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$)
e) $\sum_{i=1}^{10}\sum_{j=1}^{i}(1 + 2^j)$

Problem 3. *2 + 2 pts*

Explain the answers of following questions.

a) Let the domain and codomain of function f to be $\{a, b, c\}$. Image of a is preimage(s) of b. f is injective. What is $f(b)$? You do not have to explain.
b) With function f in (a), let function g have same domain as f and codomain $\{a, b\}$. Prove that $g \circ f$ has no inverse function.