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CS204
Fall 2017, Assignment #6

Problem 1.   *2 + 4 + 3 + 3 + 3 pts*

Define a random variable D which returns one of the numbers from $\{1, \ldots, 6\}$ uniformly, i.e., a dice.

A **dice state** is a tuple $(D_1, \ldots, D_a, b)$ where D_i are mutually independent random variables which follow the probability distribution of D, $a \in \mathbb{Z}^+, b \in \mathbb{N}$. Define S as a set of all dice state. Also call a dice state returns **critical** if at least two D_i's among all return 6; call a dice state returns **fumble** if all D_i's return 1.

A **dice result** is a function $X : S \to \mathbb{Z}^+$, which is defined as $X(D_1, \ldots, D_a, b) = \sum_{i=1}^{a} D_i + b$ for some $a \in \mathbb{Z}^+, b \in \mathbb{N}$.

Now suppose that there is a role playing game. In this game, there are **character**s who have **skill**s to attack someone and an ability called **evasion** to evade that attack.

Any skill has only one purpose: to attack someone. Each skill has its own **accuracy** where the accuracy is defined as a dice state. To judge whether a target can avoid an incoming atack, the target checks his/her **evasion**, which is defined as again a dice state.

Here are the detailed information of two characters, including their skills:

	Assassin	Monk
Skill 1 (Accuracy)	Extermination $(D_1, \ldots, D_4, 4)$	High-kick $(D_1, D_2, 8)$
Skill 2 (Accuracy)	Toxic Bomb $(D_1, D_2, D_3, 6)$	Wave Bullet $(D_1, D_2, D_3, 3)$
Evasion	$(D_1, D_2, 6)$	$(D_1, D_2, D_3, 5)$

TABLE 1. Detailed information of two characters: Assassin and Monk.

According to dice states of each skill succeeds to attack the target if at least one of the above condition holds:

(1) Accuracy dice state of the skill returns critical, evasion doesn't;
  For example, even if the Assassin makes critical on Extermination with the accuracy dice state (6, 6, 6, 6, 3), Monk can evade it with the evasion dice state (6, 6, 1, 5).

(2) Accuracy dice state of the skill doesn't return fumble, evasion does;
  For example, even Monk get a fumble (1, 1, 1, 5) as a dice state of evasion, Toxic Bomb fails if the Assassin get a fumble for the accuracy dice state (1, 1, 1, 6).

(3) None of the above conditions hold, but the accuracy dice result of the skill is strictly bigger than evasion, except the cases that the accuracy dice state returns fumble or the evasion dice state returns critical.
  For example, Assassin will successfully resist for Wave Bullet if the accuracy dice state is (2, 3, 5, 3) and the evasion dice state is (1, 6, 6); Assassin will successfully evade for High-kick even with the evasion dice state (1, 2, 6) if Monk get a fumble (1, 1, 8) on the accuracy dice state; Monk can evade with the evasion dice state (6, 1, 6, 5) if Assassin didn't get a critical on the accuracy dice state (6, 5, 5, 5, 4).

First, calculate the probability that

(a) Assassin evades the Wave Bullet;
(b) Monk fails to evade the Extermination.

Similarly, a **damage function** is also a random variable Y which is a finite linear combination of dice D with a positive constant in addition. Each skill has its own **damage function** to calculate the damage done by it. Note that some damage functions might depend on both the accuracy dice state and the evasion dice state.

Here are detailed damage functions of each skill:

Skill name	Description
Extermination	Performs $8D + 20 + (7 \times$ number of 6 occured on the accuracy dice state) damage.
High-kick	Performs $6D + 15$ damage. If the accuracy dice result was bigger than 15, performs $7D + 20$ damage instead.
Toxic Bomb	Performs $5D + 10$ damage. If the target of attack made a fumble on the evasion dice state, performs 20 damage more.
Wave Bullet	Performs $2D + 20 +$ (difference between the accuracy dice result and the evasion dice result) $\times$ 5 damage.

TABLE 2. Explanation of each skills: particularly for damage functions.

According to that, calculate

(c) the variance of the damage of Wave Bullet when the accuracy dice result was 15 whereas the evasion dice result was 10;
(d) the mean of the damage done by Toxic Bomb where the Assassin made a critical on the accuracy dice state whereas the Monk didn't on the evasion dice state;
(e) the mean of the damage done by High-kick when two dices which were used for accuracy dice state were differed by 2 whereas Assassin made a fumble on the evasion dice state.

Write your calculation process.

(If you are interested more, do other calculations by yourself. To know what this type of the game is, see https://en.wikipedia.org/wiki/Tabletop_role-playing_game*.)*

Problem 2. *3 + 3 pts*

(a) For the set $A = \{a, b, c, d\}$, there are three random variables X, Y, Z which returns 0 for two elements in A, 1 for other two elements. Construct X, Y, Z in the feasible way so that they are pairwise independent but not mutually.
(b) A discrete random variable X whose values lie in the set $\{0, 1, \dots\}$ has a **memoryless property** if for any $m, n \in \mathbb{N}$, $\mathbb{P}(X > m + n | X > m) = \mathbb{P}(X > n)$ holds. Prove that a geometric distribution with parameter p has a memoryless property.

Problem 3. *2 + 2 pts*

There are two factories A, B which publish a book. A prints 60% of the book, and B prints others. Suppose that A prints a damaged book in the 2% of probability; B prints a damaged book in the 1% of probability.

(a) Calculate the probability that a book is damaged.
(b) Suppose that a book was damaged. Calculate the probability that it was printed by A.