CS204 Discrete Mathematics, Spring 2018

Recitation #1

Time: 2017.3.8 (Thu) 19:00 ~ 19:30

TA: Donghyun Lim

1. (Rosen 1.1.37)

Construct a truth table for

$(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$.

2. (Rosen 1.1.37)

Construct a truth table for

$(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s)$.

3. Given two functions

$$f(x) = e^x, g(x) = 1 + x,$$

judge whether given propositions are true or not. If it is false, give a counterexample.

a) $\forall x\,(f(x) > 0)$

b) $\forall x\,(f(x) > g(x))$

c) $\exists x\,(f(x) \leq g(x))$

(The domain of discourse is real numbers)

4. (Rosen 1.4.52)

The notation $\exists!x\ P(x)$ denotes "There exists a unique x such that P(x) is true." If the domain consists of all integers, what are the truth values of these statements?

a) $\exists!x\,(x > 1)$

b) $\exists!x\,(x^2 = 1)$

c) $\exists!x\,(x + 3 = 2x)$

d) $\exists!x\,(x = x + 1)$

5. (1.3.77)

Show that $(p \rightarrow q) \equiv (\neg p \vee q)$ using truth table.

6.

Show that $p \rightarrow (q \wedge r)$ and $(p \rightarrow q) \wedge (p \rightarrow r)$ are logically equivalent using logical equivalences (including the one shown in problem 5). Do not use truth table.

Starting from $(p \rightarrow q) \wedge (p \rightarrow r)$, apply problem 5 twice, apply distributive law, and apply problem 5 again.

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws