CS204 Discrete Mathematics, Spring 2018

Recitation #4

Time: 2018.03.29 (Thu) 19:00 ~ 19:30

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1. Determine whether each of these functions is a bijection
from R to R.
a) f (x) = 2x + 1
b) f (x) = x^2 + 1
c) f (x) = x^3
d) f (x) = $(x^2 + 1)/(x^2 + 2)$

Sol)
a)T
b)F
c)T
d)F (both the denominator and the numerator are always positive)

2. Let f(x) = $\lfloor x^2/3 \rfloor$. Find f (S) if
a) S = {−2, −1, 0, 1, 2, 3}.
b) S = {0, 1, 2, 3, 4, 5}.
c) S = {1, 5, 7, 11}.
d) S = {2, 6, 10, 14}.

Sol)
a) {0, 1, 3}
b) {0, 1, 3, 5, 8}
c) {0, 8, 16, 40}
d) {1, 12, 33, 65}

3. Determine whether f : Z × Z → Z is onto if
a) f (m, n) = 2m − n.
*b) f (m, n) = $m^2 - n^2$
c) f (m, n) = m + n + 1.
d) f (m, n) = |m|−|n|.
e) f (m, n) = $m^2 - 4$
(Z means the set of all integers)

Sol)
a) T
b) F It is impossible to be (m+n)(m-n) = 2. If |m-n|=1, then |m+n| is odd. If |m-n|=2, then (m+n) is even.
c) T
d) T
e) F

4. Suppose that the number of bacteria in a colony triples every hour.
a) Set up a recurrence relation for the number of bacteria after n hours have elapsed.
b) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

Sol)
a) a_{n+1} = a_n * 3
b) 100 * 3^10

5. Show that the sequence {an}is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if
a) $a_n = 0$
b) $a_n = 1$
c) $a_n = (-4)^n$
d) $a_n = 2(-4)^n + 3$

Sol) omitted